ON AN ALGORITHM IN SQUARING RADIX NUMBERS: BASIS FOR A LEARNING MODULE DEVELOPMENT

IVY C. EBORA – FRONDA, PhD
https://orcid.org/0000-0003-0492-4465
ivy.ebora@deped.gov.ph
Department of Education, San Agapito National High School
Isla Verde, Batangas City, Philippines

ABSTRACT

Primarily, this study developed an algorithm in squaring radix numbers. The study is a pure research with expository as the approach. There are no statistical tools used in this study. Consultation with the experts, reading books and journals, internet resources and other reference materials were used in the conduct of the study. By using the previously accepted concepts in arithmetic and algebra, the researcher explored and was able to develop an algorithm in squaring radix numbers. The algorithm $n^2 = (n+d_0)(n-d_0)+d_0$ *where* n *is the number* d_0 *is the unit digit of* n *is developed. This can be used in squaring radix numbers*

Keywords: algorithm, learning module, radix numbers, squaring a number

INTRODUCTION

Mathematics is one subject that pervades life at any age, in any circumstance. Thus, its value goes beyond the classroom and the school. Mathematics as a subject must be learned comprehensively and with much depth. (K to 12 Curriculum Guide, 2011). Interacting with new scientific discoveries, Mathematics continues to develop when Mathematical innovations led to a rapid increase in the rate of mathematical discovery that continues to present day.

The researcher found some limitations and difficulties in using the existing techniques in squaring a number. In this sense, the researcher saw the need of having a full grasp of understanding on different Mathematical categories. Inspired by the challenges met as a secondary school Mathematics teacher, the researcher became interested in finding an innovative way in squaring a number. As a secondary school Mathematics teacher for nine (9) years, the researcher met problems in teaching squaring a number. The common mistake the learners encountered in squaring a single digit number is multiplying the base by the exponent instead of multiplying the base by itself. The researcher explored to develop an algorithm in squaring a number that will be of great help to the students and to the teachers as well.

The K to 12 Mathematics curricula is supported by the learning principles and theories: experiential and situated learning, reflective learning, constructivism, cooperative learning and discovery and inquiry-based learning. Constructivism is the theory that argues that knowledge is constructed when the learner can draw ideas from his own experiences and connects them to new ideas that are connected. This learning approach constructs explanatory models that continuously change with each new construction dependent on the available ideas and information. New patterns of frameworks are constructed by the learner to explain make sense of new events and objects that they perceived. (Bago, 2001)

Due to the changing Philippine education system, teachers need to be responsive to improve quality education. Mathematics is one of the learning areas in school; therefore, Mathematics teachers assume a significant role to improve its instruction.

OBJECTIVES OF THE STUDY

The researcher aimed to: 1) develop an algorithm for squaring radix numbers; 2) prove the developed algorithm in squaring radix numbers; and 3) develop a learning module in squaring a number with applications.

METHODOLOGY

The study is a pure research with expository as the approach. It was done for the development of an algorithm in squaring radix numbers. Using the previously accepted concepts in arithmetic and algebra, the researcher explored and developed an algorithm in squaring r-digit numbers. Books, internet resources, compilation of notes and consultation with experts served as references of the study.

More so, it was also done for the development of a learning module in squaring a number. A learning module is a packet of teaching materials consisting of behavioral objectives, a sequence of learning activities and provisions for evaluation. (Rishekesh,S., 2015)

RESULTS AND DISCUSSION

1. Development of an Algorithm for Squaring Radix Numbers

Algorithm A for calculating the square of any decimal number that ends in 5 exists.
Let n be an $(r+1)$-digit decimal number that ends in the unit digit (5), so that

$$n = d_r d_{r-1} \ldots d_2 d_1 d_0 \; \mathrm{n} = \mathrm{d_r d_{r-1} \ldots d_2 d_1 d_0}$$

where $\mathrm{d}_0 = 5$.

Writing n in the form $n = d_r d_{r-1} \ldots d_2 d_1 d_0 \cdot 10 + 5$ and with $m = d_r d_{r-1} \ldots d_2 d_1$, we obtain, on squaring a number $n^2 = (m \cdot 10 + 5)^2$

$$= m^2 \cdot (10)^2 + m \cdot (10)^2 + 25$$

$$n^2 = m(m+1)(10)^2 + 25. \mathrm{n}^2 = (\mathrm{m} \cdot 10 + 5)^2$$

$$= \mathrm{m}^2 \cdot (10)^2 + \mathrm{m} \cdot (10)^2 + 25$$

This formula, which inductively holds for all-natural numbers m, forms the basis for the following simple algorithm for calculating the square of any decimal numbers that ends in 5.

1.1 Algorithm A

1. Square the unit digit f 5 of n. (nnis the decimal number that ends in 5)
2. Remove the unit digit 5 from nn and denote the resulting decimal number as m.
3. Compute $m(m+1)$.
4. Append 25 (the square of the unit digit 5 obtained in step 1) to the right end of the number computed in step 3 and stop. [the resulting number is n^2]

Examples

(a) $(25)^2 = 625$ [$6 = 2 \cdot 3$, and 25 (= 5^2) is appended to the right end of 6]

(b) $(75)^2 = 5625$ [$56 = 7 \cdot 8$, and 25 (= 5^2) is appended to the right end of 56]

(c) $(1005)^2 = 1010025$ [$10100 = 100 \cdot 101$, and 25 (= 5^2) is appended to the right end of 10100]

The Russian Peasant algorithm

Let a and c be given two decimal numbers whose product is to be calculated. If you write one of the decimal numbers, say a a, in binary form, $\mathrm{b_i} \in \{0,1\}$ $b_i \in \{0,1\}$ then form

$$c \cdot a = c \cdot (b_r \cdot 2^r + b_{r-1} \cdot 2^{r-1} + \ldots + b_1 \cdot 2^1 + b_0 \cdot 2^0)$$

$$= (c \cdot 2^r) \cdot b_r + (c \cdot 2^{r-1}) \cdot b_{r-1} + \ldots + (c \cdot 2^1) \cdot b_1 + (c \cdot 2^0) \cdot b_0$$

it is seemed that the product $c \cdot a$ has been transformed into a sum of terms of the form $c \cdot 2^i$-b_i, with

$$c \cdot 2^i - b_i = \begin{cases} 0 \\ c \cdot 2^i \end{cases} \text{ if } b_i = 0,\ b_i = 1 \text{ for } i = 0, \ldots, r.$$

1.2 Algorithm B (Russian Peasant algorithm)

1. Write down a and c (a is to be the starting member of a left column of numbers, and c is to be the starting member of the corresponding right column of numbers)
2. Divide a by 2, and record only the quotient a_0 of this division below a. Then multiply c by 2, and record the result $c_0\{c_0 = 2c\}$ $\{c_0 = 2c\}$ below c.
3. For $i = 1$ to $i = r\text{-}1$ (r is the smallest positive integer for which the division of a by 2^{r+1} produces the quotient $a_r = 0$),divide a_{i-1} by 2 and record only the quotient a_i obtained below a_{i-1} Then multiply c_{i-1} by 2, and write the result $c_i\{c_i = 2c_{i-1}\}$ $\{c_i = 2c_{i-1}\}$below c_{i-1}.
4. Cross out all the even numbers in the left column along with the corresponding numbers in the right column.
5. Add the numbers in the right column, which were not crossed out in step 4, and stop. (This sum is the product of $a \cdot c$.)

To calculate $(6875)^2$ we proceed as follows:

1. Square the unit digit 5 to obtain 25.
2. Calculate 688 x 687, using the Russian Peasant algorithm:

~~688~~	~~687~~
~~312~~	~~1374~~
~~172~~	~~2748~~
~~86~~	~~5496~~
43	10992
21	21984
~~10~~	~~43968~~
5	87936
~~2~~	~~175872~~
1	351744
	472656

{688 x 687 = 472656}

3. Append 25 {obtained in step 1} to the right end of 472656.

$\{(688)^2 = 47265625\}$

Let $(n)_b$ be a number in some base b that ends in the unit digit $(u)_b = \frac{b}{2}, (u)_b = \frac{b}{2}, b \geq 2$ being necessarily an even decimal number. Then $(n)_b$ can be written in the form $(n)_b = (m)_b \cdot (10)_b + (u)_b = (m)_b \cdot b + \frac{b}{2}$ where m is obtained from n by deleting the unit digit of n.

Squaring, we obtain

$$(n^2)_b = (m^2)_b \cdot b^2 + (m)_b \cdot b^2 + \left(\frac{b}{2}\right)^2$$

$$= ((m)_b \cdot (m+1)_b) \cdot b^2 + \left(\frac{b}{2}\right)^2$$

Since $b^2 = (100)_b$, calculating $((m)_b \cdot (m+1)_b) \cdot b^2$ in the base b is equivalent to appending two 0s to the number $m(m+1)$ in the base b, noting that 0 belongs to all bases. Hence, Algorithm A, when suitably modified can also be used for squaring numbers in a base b, that end in the unit digit $(u)_b = \frac{b}{2}$ where $b \geq 2$ is an even decimal number.

1.3 Algorithm C

1. Square the unit digit $(u)_b$ of $(n)_b$ $\{(u)_b = \frac{b}{2}; b$ is an even decimal number}
2. Remove the unit digit $(u)_b$ from $(n)_b$ and denote resulting member as $(m)_b$.

3. Compute $(m_b) \cdot (m+1)_b$ {base-b arithmetic}

4. Append $((u)_b)^2$ {$((u)_b)^2$ $((u)_b)^2$ must be in the form of a 2-digit number in base b} to the right end of the number computed in step 3, and stop. {The resulting number is $((n)_b)^2$.}

When the base $b = 2^n (n \geq 1)$, Algorithm C can be used for squaring numbers in that base, that ends in 2^{n-1}.

Algorithm C is used to compute the squares of all binary numbers that end in the bit 1, i.e., of all odd decimal numbers. Note that in this case, the binary number 1^2 to be appended to the binary number $m(m+1)$ in step 5 of the algorithm is to be $(01)_2$.

Examples

(a) The decimal number $n = 89$ has the binary representation 1011001. Using Algorithm C, we obtain

$$
\begin{aligned}
((1011001)_2)^2 &= ((101100)_2 \cdot (10)_2 + 1)^2 \\
&= (101100)_2 \cdot (101101)_2 + 1 \\
&= (11110111100)_2 \cdot (100)_2 + (01)_2 \\
&= (1111011110001)_2 \\
&= 7921 \text{ which is } (89)^2.
\end{aligned}
$$

(b) The decimal number $n = 324$ has 101 000 100 as its binary representation so that its octal representation is $(504)_8$. To square $(504)_8$, use Algorithm C to obtain

$$
\begin{aligned}
((504)_8)^2 &= ((50)_8 \cdot (10)_8 + 4)^2 \\
&= (50)_8 \cdot (51)_8 \cdot (100)_8 + 4^2 \\
&= (3150)_8 \cdot (100)_8 + (20)_8 \\
&= (315020)_8 \\
&= 104976,
\end{aligned}
$$

which is $(324)^2$, noting that $(315020)_8 = (11\ 001\ 101\ 000\ 010\ 000)_2$.

(c) The decimal number $n = 168$ has 1010 1000 as its binary representation, so that its hexadecimal representation is $(A8)_{hex}$. Squaring $(A8)_{hex}$, using Algorithm C, you obtain

$$
\begin{aligned}
((A8)_{hex})^2 &= ((A)_{hex} \cdot (10)_{hex} + 8)^2 \\
&= (A(A+1)_{hex} \cdot (100)_{hex} + 8^2 \\
&= (6E00)_{hex} + (40)_{hex} \\
&= (6E40)_{hex} \\
&= 28224
\end{aligned}
$$

which is $(168)^2$, noting that $(6E40)_{hex} = (110\ 1110\ 0100\ 0000)_2$.

In example (a) when you remove the rightmost bit 1 from the binary representation of an odd number *n*, call the resulting binary number by m,and multiply *m* by $(10)_2$, i. e. by the decimal number 2, then transform *n* into its (even) predecessor. Hence, what will be obtained when Algorithm C terminates is

$$
\begin{aligned}
n^2 &= ((n-1)+1)^2 \\
&= (n-1)^2 + 2(n-1) + 1 \\
&= (n-1)(n+1) + 1 \quad \text{equation (1)}
\end{aligned}
$$

But equation (1) is an identity for any number n, whether n is odd or even, or a number in any base. Therefore, equation (1) can be used as a basis for computing the square of a number n in any base b.

But the presented algorithm for decimal numbers is limited to numbers ending in 5. The researcher developed an algorithm for squaring decimal numbers from two-digit to the *r*-digit.

Let n be a $(10x + y)$-digit decimal number, so that $n = 10x + y$; where $0 \leq y \leq 9$. Writing *n* in the form $n = 10x + (1 \leq y \leq 9)$, $n = 10x + (1 \leq y \leq 9)$, you obtain on squaring

$$
\begin{aligned}
n^2 &= (10x + y)^2 \\
&= (10x)^2 + 2(10x)(y) + (y)^2 \\
&= 100x^2 + 20xy + y^2
\end{aligned}
$$

Examples

$$
\begin{aligned}
(19)^2 &= (10 * 1 + 9)^2 \\
&= (10)^2(1)^2 + 2(10)(1)(9) + (9)^2 \\
&= 100 + 180 + 81 \\
&= 361
\end{aligned}
$$

$$(34)^2 = (10 * 3 + 4)^2$$
$$= (10)^2(3)^2 + 2(10)(3)(4) + (4)^2$$
$$= 900 + 240 + 16$$
$$= 1\ 156$$

$$(56)^2 = (10 * 5 + 6)^2$$
$$= (10)^2(5)^2 + 2(10)(5)(6) + (6)^2$$
$$= 2500 + 600 + 36$$
$$= 3\ 136$$

This is also true in squaring a three-digit number.

Let n be a $(100x + 10y + z)$-digit decimal number, so that $n = 100x + 10y + z$; $(100x + 10y + z)$ where $0 \leq z \leq 9$. Writing n in the form $(100x + 10y + z)n = 100x + 10y + (1 \leq y \leq 9)$, you obtain on squaring

$$n^2 = (100x + 10y + z)^2$$
$$= (100x)^2 + (10y)^2 + z^2 + 2(100x)(10y) + 2(100x)(z) + 2(10y)(z)$$
$$= 1000x^2 + 100y^2 + z^2 + 2000xy + 2000xz + 20yz$$

Using the same pattern in squaring a three-digit number: $n^2 = (100x + 10y + z)^2$

$$(199)^2 = (100*1 + 10*9 + 9)^2$$
$$= (100 + 90 + 9)^2$$
$$= (100)^2 + (90)^2 + (9)^2 + 2(100)(90) + 2(100)(9) + 2(90)(9)$$
$$= 10\ 000 + 8\ 100 = 81 + 18\ 000 + 1\ 800 + 1\ 620$$
$$= 39\ 601$$

Based from the above examples, the researcher explored in squaring decimal numbers from two-digit to r-digit and developed a new algorithm for squaring decimal numbers.

Observation 1:

To square two-digit numbers

$$99^2 = (99+9)(99-9)+(9)^2$$
$$= (108)(90)+81$$
$$= 9720+81$$
$$= 9801$$

To square 56,

$$56^2 = (56+6)(56-6)+(6)^2$$
$$= (62)(50)+36$$
$$= 3100+36$$
$$= 3136$$

Observation 2:

Squaring a three-digit number,

$$199^2 = (199+9)(199-9)+(9)^2$$
$$= (208)(190)+81$$
$$= 39520+81$$
$$= 39601$$

Observation 3:

To square a four-digit number:

$$1234^2 = (1234+4)(1234-4)+(4)^2$$
$$= (1238)(1230)+16$$
$$= 1522740+16$$
$$= 1522756$$

Observation 4:

To square a five-digit number:

$$12345^2 = (12345+5)(12345-5)+(5)^2$$
$$= (12350)(12340)+25$$
$$= 152399000+25$$
$$= 152399025$$

Observation 5:

In squaring a six-digit number,

$$123456^2 = (123456+6)(123456-6)+(6)^2$$
$$= (123462)(123450)+36$$
$$= 1524138390+36$$
$$= 1524138426$$

Observation 6:

Squaring a seven-digit number,

$$1234567^2 = (1234567+7)(1234567-7)+(7)^2$$
$$= (1234574)(1234560)+49$$
$$= 1524155677440+49$$
$$= 1524155677489$$

Observation 7:

To square an eight-digit number

$$\begin{aligned}12345678^2 &= (12345678+8)(12345678-8)+(8)^2\\ &= (12345686)(12345670)+64\\ &= 152415765279620+64\\ &= 152415765279684\end{aligned}$$

Based on the algorithm in squaring a two-digit number to an eight-digit number, the following algorithm is derived for squaring an r-digit number.

Let n be a decimal number, so that $n = 10^r d_r + 10^{r-1} d_{r-1} + \cdots + 10^2 d_2 + 10 d_1 + 10^0 d_0$ where $0 \leq d_0 \leq 9$. Writing n in the form $n = 10^r d_r + 10^{r-1} d_{r-1} + \ldots + 10^2 d_2 + 10 d_1 + 10^0 d_0$, we obtain on squaring, $n^2 = (n + d_0)(n - d_0) + (d_0)^2$ where n is the number and d_0 is the unit digit of n.

This formula, which inductively holds for all-natural numbers, n forms the basis for the following algorithm for calculating the square of any r-digit decimal numbers.

1. Add the number n by its unit digit d_0 and denote the sum as p
2. Subtract the unit digit d_0 from the number n and denote the difference as q q.
3. Multiply p and q.
4. Square the unit digit d_0.
5. Add the product in step 3 and the result in step 4. The resulting number is n^2.

2. Proof of the Developed Algorithm for Squaring Radix Numbers

Let n be a decimal number, so that $n = 10^r d_r + 10^{r-1} d_{r-1} + \cdots + 10^2 d_2 + 10 d_1 + 10^0 d_0$ where $0 \leq d_0 \leq 9$. Writing n in the form $n = 10^r d_r + 10^{r-1} d_{r-1} + \ldots + 10^2 d_2 + 10 d_1 + 10^0 d_0$, we obtain on squaring, $n^2 = (n + d_0)(n - d_0) + (d_0)^2$ where n is the number and d_0 is the unit digit of n.

Algorithm for squaring an r-digit number

1. Add the number n by its unit digit d_0 and denote the sum as p
2. Subtract the unit digit d_0 from the number n and denote the difference as q q.
3. Multiply p and q.
4. Square the unit digit d_0.
5. Add the product in step 3 and the result in step 4. The resulting number is n^2.

To prove the developed algorithm in squaring *r-digit* numbers, illustrative examples are presented.

$$\begin{aligned}37^2 &= (37+7)(37-7)+(7)^2\\ &= (44)(30)+49\\ &= 1320+49\\ &= 1369\end{aligned}$$

To find the square of 326,

$$\begin{aligned}326^2 &= (326+6)(326-6)+(6)^2\\ &= (332)(320)+36\\ &= 106240+36\\ &= 106276\end{aligned}$$

To square 3 219,

$$\begin{aligned}3219^2 &= (3219+9)(3219-9)+(9)^2\\ &= (3228)(3210)+81\\ &= 10361880+81\\ &= 10361961\end{aligned}$$

To square 52 876,

$$\begin{aligned}52876^2 &= (52876+6)(52876-6)+(6)^2\\ &= (52882)(52870)+36\\ &= 2795871340+36\\ &= 2795871376\end{aligned}$$

In squaring 402 753,

$$\begin{aligned}402753^2 &= (402753+3)(402753-3)+(3)^2\\ &= (402756)(402750)+9\end{aligned}$$

$$= 162209979000 + 9$$
$$= 162209979009$$

Squaring 8 112 635,

$$8112635^2 = (8112635+5)(8112635-5)+(5)^2$$
$$= (8112640)(8112630)+25$$
$$= 65814846643200+25$$
$$= 65814846643225$$

To square 78 924 631,

$$78924631^2 = (78924631+1)(78924631-1)+(1)^2$$
$$= (78924632)(78924630)+1$$
$$= 617556043286160+1$$
$$= 617556043286161$$

Based from the above examples, the researcher proved that to square any r-digit number n, the derived algorithm is new and is functional. In squaring any r-digit decimal number, the formula $n^2 = (n + d_0)(n-d_0) + (d_0)^2$ where n is the number and d_0 is the unit digit of n can be utilized.

3. Development of a Learning Module

The learning module is intended for Grade 6 to Grade 10 students. It includes discussion and examples on various topics involving square of a number and provides exercises to improve the learner's skills in squaring a number using the developed algorithm instead of using a calculator.

Design. The researcher developed the rationale and objectives of the learning module. The K to 12 Mathematics curricula was reviewed to identify the topics and learning competencies in Mathematics wherein squaring a number is used. Learning outcomes were also defined upon reviewing the curriculum guide.

Master teachers and colleagues at Batangas National High School Mathematics department were consulted for the enhancement of the module. Comments and suggestions of the researcher's adviser and panel of examiners helped to improve the learning module.

Each topic was clearly identified with its learning competency. Various examples in squaring *r*-digit numbers are presented for the learners' reference in answering the exercises. Furthermore, solving problems involving squaring a number is included in the module. The learners may use the developed algorithm in answering problem.

4. The Learning Modules

4.1 Squaring a Number

Competency: The learner applies the laws of exponents in squaring a number

Algorithm for squaring an r-digit number

1. Add the number n by its unit digit d_0 and denote the sum as p
2. Subtract the unit digit d_0 from the number n and denote the difference as q q.
3. Multiply p and q.
4. Square the unit digit d_0.
5. Add the product in step 3 and the result in step 4. The resulting number is n^2.

Examples:

$$37^2 = (37+7)(37-7)+(7)^2$$
$$= (44)(30)+49$$
$$= 1320+49$$
$$= 1369$$

$$326^2 = (326+6)(326-6)+(6)^2$$
$$= (332)(320)+36$$
$$= 106240+36$$
$$= 106276$$

4.2 Square of a Fraction

Competency: The learner applies the laws of exponents in squaring a fraction

Power of a Quotient

If a and b are any real numbers, b ≠ 0, and n is a positive integer, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Examples

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

$$\left(\frac{1}{4}\right)^2 = \frac{1^2}{4^2} = \frac{1}{16}$$

4.3 Product of the Square of a Binomial

Competency: The learner uses models and algebraic methods to find the square of a binomial

Pattern: $(a+b)^2 = a^2 + 2ab + b^2$

- The product of the square of a binomial is a perfect square trinomial.

$$(a+b)^2 = (a+b)(a+b)$$
$$(a-b)^2 = (a-b)(a-b)$$
$$= a(a) + a(b) + b(a) + b(b)$$
$$= a(a) + a(-b) + (-b)(a) + (-b)(-b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 + 2ab + b^2$$
$$= a^2 - 2ab + b^2$$

Examples

Solution 1

1. $(x+2)^2 = (x+2)(x+2)$
$$= x(x) + x(2) + 2(x) + 2(2)$$
$$= x^2 + 2x + 2x + 4$$
$$= x^2 + 4x + 4$$

Solution 2

$$(x+2)^2 = (x)^2 + 2(2)(x) + (2)^2$$
$$= x^2 + 4x + 4$$

2. $(x-5)^2 = (x-5)(x-5)$
$$= x(x) + x(-5) + (-5)(x) + (-5)(-5)$$
$$= x^2 + (-5x) + (-5x) + 25$$
$$= x^2 - 10x + 25$$

Solution 2

$$(x-5)^2 = (x)^2 + 2(-5)(x) + (-5)^2$$
$$= x^2 - 10x + 25$$

3. $(3x+7)^2 = (3x)^2 + 2(7)(x) + (7)^2$
$$= 9x^2 + 14x + 49$$

4.4 Product of the Sum and Difference of Two Terms

Competency: The learner uses models and algebraic methods to find the product of the sum and difference of two terms

Pattern: $(a+b)(a-b) = a^2 - b^2$

- The product of the sum and difference of two terms is a difference of two squares.

Examples:

1. $(x+14)(x-14) = (x)^2 - (14)^2$
$$= x^2 - 196$$

2. $(4x^3+21)(4x^3-21) = (4x^3)^2 - (21)^2$
$$= 16x^6 - 441$$

4.5 Area of a Square

Competency: The learner finds the area of a square and solves problems involving area of a square

A square is a rectangular quadrilateral, which means that it has four equal sides and four equal angles (90-degree angles or right angles). It can also be defined as a rectangle in which two adjacent sides have equal length. To find the area of a square, use the formula $A = s^2$ where A is the area and s is the measure of its sides.

Example

Find the area of a square which side measures 15 centimeters.

Given:

$s = 15$ cm

Solution:

$$\begin{aligned} A &= s^2 \\ &= (15)^2 \text{ cm} \\ &= (15 + 5)(15 - 5) + (5)^2 \\ &= (20)(10) + 25 \\ &= 200 + 25 \\ &= 225 \text{ cm}^2 \end{aligned}$$

4.6 Area of a Circle

Competency: The learner finds the area of a circle and solves problems involving circles

A circle is a closed curve formed by a set of points on a plane that are the same distance from its center. The area of a circle is the region enclosed by the circle. The area of a circle is equals to pi (π) multiplied by its radius squared. Pi (π) is the ratio of the circumference of a circle to its diameter. The area of a circle is given by the formula: $A = \pi r^2$; where A is the area and r is the radius.

Example

Find the area of a circle with a radius of 26 centimeters.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(26)^2 \\ &= \pi\left[(26+6)(26-6)+6^2\right] \\ &= \pi\left[(32)(20)+36\right] \\ &= \pi(640+36) \\ A &= 676\pi cm^2 \end{aligned}$$

4.7 Surface Area of a Sphere

Competency: The learner finds the surface area of a sphere and solves problems involving surface area of a sphere

A sphere is a solid in which all the points on the round surface are equidistant from a fixed point, known as the center of the sphere. The distance from the center to the surface is the radius. Surface area of a sphere is given by the formula:

Surface Area of sphere = $4\pi r^2$; where r is the radius of the sphere.

Example

Calculate the surface area of a sphere with radius 32 centimeters.

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(32)^2 \\ &= 4\pi\left[(32+2)(32-2)+2^2\right] \\ &= 4\pi\left[(34)(30)+4\right] \\ &= 4\pi(1020+4) \\ &= 4\pi(1024) \\ S &= 4096\pi cm^2 \end{aligned}$$

4.8 Surface Area of Solid Figures

Competency: The learner finds the surface area of a solid figure and solves problems involving surface area of solid figures

The surface area of a solid figure is the sum of the areas of all faces of the figure.

Example

Find the surface area of the cone if the radius of its circular base is 23 cm and if the slant height measures 7 cm.

$$
\begin{aligned}
SA_{cone} &= 2pr^2 + 2prh \\
&= 2p\pi(23)^2 + 2p(23)(7) \\
&= 2p[(23+3)(23-3)+3^2] + 2p(161) \\
&= 2p[(26)(20)+9] + 322p \\
&= 2p(520+9) + 322p\pi \\
&= 2\pi(529) + 322\pi \\
&= 1\,058\pi + 322\pi \\
SA_{cone} &= 1\,380\pi \text{ cm}^2
\end{aligned}
$$

CONCLUSIONS

Based on the objectives of the study, the following conclusions were drawn:

1. The researcher developed an algorithm for squaring radix numbers specifically the algorithm for squaring decimal numbers form two-digit to r-digit.
2. The new developed algorithm was proved to be functional and can be utilized by students and teachers.
3. A learning module was crafted in squaring a number with applications.

RECOMMENDATIONS

For the enhancement of the study, the following recommendations were endorsed:

1. Explore on squaring a number using other techniques
2. Conduct a research on raising a number to a power greater than two (2).

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AUTHOR'S PROFILE

Ivy C. Ebora – Fronda, an alumna of the University of Batangas where she finished Bachelor of Secondary Education major in Mathematics (CumLaude), Master of Science in Mathematics and Doctor of Philosophy in Education major in Mathematics. She was a Secondary School Mathematics Teacher from May 2007 to April 2010 at Sta. Teresa College, Bauan, Batangas, Philippines. From June 2010 to July 2018 she served as a Mathematics teacher at Batangas National High School, Batangas City. She became an International Leaders in Education Program (ILEP) Fellow at Clemson University, South Carolina, USA in January to May 2014. ILEP is a program sponsored and funded by the U.S Department of State and International Exchange (IREX). At present, she is a secondary school principal II at San Agapito Natonal HIgn SchooL, Isla Verde, Batangas City. She is a servant leader who aims to render service wholeheartedly to her Alma Mater and fellow Verdenians to the best of her ability.

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