AN EXPLORATION OF SYNTHETIC DIVISION FOR NON-LINEAR POLYNOMIAL DIVISORS

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ABSTRACT

This paper is an exploration of the synthetic division in compact form. The main goal was to develop an alternative algorithm on division of polynomials whose dividend is $P(x) = a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \cdots + a_nx + a_{n+1}$ and the divisor is $D(x) = b_1x^m + b_2x^{m-1} + b_3x^{m-2} \ldots + b_mx + b_{m+1}$, where $n > m$, $a_1 \neq 0$, $b_1 \neq 0$, and $a_i's$ and $b_i's$ including a_{n+1} and b_{m+1} are constant. The quotient is in the form $Q(x) = q_1x^{n-m} + q_2x^{n-m-1} + q_3x^{n-m-2} + \cdots + q_{n-m}x + q_{n-m+1}$ while the remainder is $R(x) = r_1x^{m-1} + r_2x^{m-2} + \cdots + r_{m-1} + r_m$. Specifically, this aimed to develop an algorithm using synthetic division in compact arrangement when b_1=1 and b_1≠1, to provide additional conditions to the problems where the usual synthetic divisions are inappropriate, and to generalize the developed algorithm of dividing polynomials of higher degrees. Basic research was employed in this study. This resulted to the development of alternative algorithm initially for quadratic, cubic and quartic divisors. Also, the conditions on inappropriateness of the usual synthetic division were identified. To wit, (a) the divisor $D(x) = b_1x^m + b_2x^{m-1} + b_3x^{m-2} + \cdots + b_mx + b_{m-1}$ is prime or irreducible; (b) in case $D(x)$ is reducible, at least one of the factors of $D(x)$ is irreducible non-linear polynomial; and (c) in case $D(x)$ is reducible and its factors are all linear polynomial (either monic or non-monic), one of them is not a factor of $P(x)$. As a generalized result, the quotient is formulated as $Q(x) = q_1x^{n-m} + q_2x^{n-m-1} + q_3x^{n-m-2} + \cdots + q_{n-m}x + q_{n-m+1}$ where $q_n = a_n - b_2q_{n-1} - \cdots - b_{m+1}q_{n-m}$ if $b_1 = 1$ or $q_n = \frac{a_n - b_2q_{n-1} - \cdots - b_{m+1}q_{n-m}}{b_1}$ if $b_1 \neq 1$ while the remainder is $R(x) = r_1x^{m-1} + \cdots + r_{m-3}x^3 + r_{m-2}x^2 + r_{m-1}x + r_m$ where $r_1 = q_{n-m+2}$, $r_2 = q_{n-m+3}$, $r_3 = q_{n-m+4}, \ldots, r_m$. Hence, the results comprise algorithms, conditions and generalized formula that can be used as alternative method in dividing polynomials with non-linear polynomial divisors.

Keywords: Mathematics education, Synthetic division, Basic research, Philippines